## Deliberation and the Wisdom of Crowds

## Supplementary Material: Code for Simulations

## For Journal Website

For each parameter constellation under consideration, our Monte Carlo simulation generates many independent outcomes of the pre- and post-deliberation opinion structure, and then derives estimates of relevant quantities by taking appropriate averages across simulation rounds. The simulations were run in Python 3. We first coded an explicit routine that represented the opinion structure in lists (not shown here). To validate and to improve speed, we took a different coding approach by vectorizing with numpy arrays. This speed-optimized, documented code is shown below.

```
import numpy as np
from itertools import combinations
rng = np.random.default rng()
def share_absorb(rounds, n, S_size, sd, p_acc, p_shares, p_absorbs):
    """Monte\ Carlo\ simulation\ with\ rounds\,,\ group\ size\,,\ source\ size\,,
    standard deviation, and the access, shareing, and receiving parameters """
    \# list to track pre-deliberation majority
    pre result tracker = []
    \# list to track post-deliberation majority
    post result tracker = []
    \# list to track pre-deliberation spread imbalance
    si tracker = []
    \# list to track post-deliberation spread imbalance
    si plus tracker = []
    \#\ list\ to\ track\ pre-deliberation\ interpersonal\ imbalance
    ii tracker = []
    \#\ list\ to\ track\ post-deliberation\ interpersonal\ imbalance
    ii plus tracker = []
    for r in range(rounds):
        \# create boolean array with access S i based on access parameter
        \# Each [i,s] entry is whether individual i has access to source s
        access = p_acc - np.random.rand(n, S_size) > 0
        \# \ calculate \ spread \ imbalance \ index
        si sum = 0
        \# for all spread pairs (combination), for each count spread on axis 0
        for tup in combinations (np.sum(access, axis=0), 2):
            #calculate spread imbalance
```

```
s1 = abs(tup[0] - tup[1])
    s2 = (tup[0] + tup[1]) / 2
    \#sum up if denominator is not 0 (otherwise assume result is 0)
    \mathbf{i} \mathbf{f} \mathbf{s} 2 != 0 :
        si sum += s1/s2
si\_sum = si\_sum * 2 / (S\_size * (S\_size-1))
si\_tracker.append(si\_sum)
\#\ create\ evidences\ based\ on\ normal\ distribution
e_s = rng.normal(1, scale = sd, size = S_size)
# limit evidences to each individual with access only
\# each [i,s] entry is the evidence individual i has from source s
\mathbf{e} \ \mathbf{s} \ \mathbf{i} = \mathbf{e} \ \mathbf{s} \ * \ \mathbf{access}
\# calculate interpersonal imbalance index pre-deliberation
ii \quad sum = 0
# for all individual evidence pairs (combination)
# calculate absolute total evidence by summing an axis 1
for tup in combinations (abs(np.sum(e_s_i, axis=1)), 2):
    \#calculate\ interpersonal\ imbalance\ index
    s1 = abs(tup[0] - tup[1])
    s2 = (tup[0] + tup[1]) / 2
    \#sum\ up\ if\ denominator\ is\ not\ 0
    \mathbf{i} \mathbf{f} \mathbf{s} 2 != 0 :
        ii sum += s1/s2
ii_sum = ii_sum * 2 / (n * (n-1))
ii_tracker.append(ii_sum)
\# votes as the sum of evidences each individual has access to
pre\_votes = np.sign(np.sum(e\_s\_i, axis=1)) # per individual
pre_result = np.sign(np.sum(pre_votes)) # as group
{\tt pre\_result\_tracker.append(pre\_result)}
\# Sharing
# check who reaches random prob to share,
# but only among those with access
who shares = access * (p shares - np.random.rand(n, S size) > 0)
# determine which sources have been shared at least once
sent = np.any(who\_shares, axis=0)
# determine who reaches random prob to absorb,
\# \ but \ only \ among \ the \ sources \ shared
# join these new links with the existing links
S\_i\_plus = sent * (p\_absorbs - np.random.rand(n, S\_size) > 0) + access
# evidences for each individual with post-deliberation sources
e \ s \ i \ plus = e \ s * S \ i \ plus
\# take post-deliberation votes
post votes = np.sign(np.sum(e s i plus, axis=1))
post result = np. sign(np.sum(post votes))
post result tracker.append(post result)
\# calculate post-deliberation spread imbalance index
si sum = 0
for tup in combinations (np.sum(S i plus, axis=0), 2):
    s1 = abs(tup[0] - tup[1])
    s2 = (tup[0] + tup[1]) / 2
    if s2 !=0 :
        si sum += s1/s2
si sum = si sum * 2 / (S size * (S size - 1))
si plus tracker.append(si sum)
```

```
\#\ calculate\ post-deliberation\ interpersonal\ imbalance\ index
                  ii \quad sum = 0
                   for tup in combinations (abs(np.sum(e_s_i_plus, axis=1)), 2):
                                    s1 = abs(tup[0] - tup[1])
                                    s2 = (tup[0] + tup[1]) / 2
                                    if s2 !=0 :
                                                   ii\_sum += s1/s2
                  ii\_sum = ii\_sum * 2 / (n * (n-1))
                  ii\_plus\_tracker.append(ii\_sum)
\# summarize aggregate estimates
predelib\_comp = \mathbf{sum}([\mathbf{x}\!\!=\!\!-1 \ \mathbf{for} \ \mathbf{x} \ \mathbf{in} \ pre\_result\_tracker]) \ / \ rounds
postdelib\_comp = sum([x==1 for x in post\_result\_tracker]) / rounds
si mean = np.mean(si tracker)
si_plus_mean = np.mean(si_plus_tracker)
ii\_mean = np.mean(ii\_tracker)
ii\_plus\_mean = np.mean(ii\_plus\_tracker)
comp\_diff = postdelib\_comp - predelib\_comp
si_change = (si_plus_mean - si_mean) / si_mean
ii_change = (ii_plus_mean - ii_mean) / ii_mean
\textbf{return} \hspace{0.1cm} (\hspace{0.1cm} predelib\_comp \hspace{0.1cm}, \hspace{0.1cm} postdelib\_comp \hspace{0.1cm}, \hspace{0.1cm} si\_mean \hspace{0.1cm}, \hspace{0.1cm} si\_plus\_mean \hspace{0.1cm}, \hspace{0.1cm} ii\_mean \hspace{0.1cm}, \hspace{0.1cm} all plus\_mean 
                                    ii_plus_mean, comp_diff, si_change, ii_change)
```